

Calcoliamo

$$\int \frac{x^3}{x^8 + 5} dx.$$

Si ha

$$\int \frac{x^3}{x^8 + 5} dx = \frac{1}{4} \int \frac{4x^3}{x^8 + 5} dx = \frac{1}{4} \int \frac{4x^3}{(x^4)^2 + 5} dx.$$

Ponendo $x^4 = t$ si ottiene $4x^3 dx = dt$ e quindi

$$\begin{aligned} \frac{1}{4} \int \frac{dt}{t^2 + 5} &= \frac{1}{20} \int \frac{dt}{\frac{t^2}{5} + 1} = \frac{1}{20} \int \frac{dt}{\left(\frac{t}{\sqrt{5}}\right)^2 + 1} \\ &= \frac{\sqrt{5}}{20} \int \frac{\frac{1}{\sqrt{5}}}{\left(\frac{t}{\sqrt{5}}\right)^2 + 1} dt = \frac{\sqrt{5}}{20} \arctan\left(\frac{t}{\sqrt{5}}\right) + C \\ &= \frac{1}{4\sqrt{5}} \arctan\left(\frac{x^4}{\sqrt{5}}\right) + C. \end{aligned}$$