Calcoliamo

$$\int \frac{2x+1}{x^4+3x^3+4x^2+3x+1}dx.$$

Il denominatore della integranda si scompone nel seguente modo:

$$x^4+3x^3+4x^2+3x+1 = (x+1)^2(x^2+x+1).$$

Usiamo la formula di Hermite per decomporre l'integranda:

$$\begin{aligned}
&\frac{2x+1}{x^4+3x^3+4x^2+3x+1} = \frac{ax+b}{x^2+x+1} + \frac{c}{x+1} + \frac{d}{(x+1)^2} \\
= \; &\frac{(a+c)x^2 + (2a+b+2c+d)x^2 + (a+2b+2c+d)x + (b+c+d)}{(x+1)^2(x^2+x+1)}
\end{aligned}$$

$$\begin{cases} a+c=0 \\ 2a+b+2c+d=0 \\ a+2b+2c+d=2 \\ b+c+d=1 \end{cases} \qquad \begin{cases} a=-1 \\ b=1 \\ c=1 \\ d=-1 \end{cases}$$

E dunque

$$\begin{aligned}
I &= \int \frac{1-x}{x^2+x+1}dx + \int \frac{1}{x+1}dx - \int \frac{1}{(x+1)^2}dx \\
&= -\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx + \frac{3}{2}\int \frac{1}{x^2+x+1}dx + \log|x+1| + \frac{1}{x+1} + c \\
&= -\frac{1}{2}\log(x^2+x+1) + \frac{3}{2}\int \frac{1}{(x+\frac{1}{2})^2+\frac{3}{4})}dx + \log|x+1| + \frac{1}{x+1} + c \\
&= -\frac{1}{2}\log(x^2+x+1) + \sqrt{3}\arctan\frac{2}{\sqrt{3}}(x+\frac{1}{2}) + \log|x+1| + \frac{1}{x+1} + c
\end{aligned}$$