Calcoliamo

$$\int \frac{1}{\sin x + \cos x} dx.$$

Dalle formule

$$\sin x = \frac{2\tan x/2}{1+\tan^2 x/2} \quad \cos x = \frac{1-\tan^2 x/2}{1+\tan^2 x/2}$$

poniamo $\tan x/2 = t$ da cui

$$dx = \frac{2}{1+t^2} dt.$$

Si ha allora

$$\begin{aligned}
\int \frac{1}{\sin x + \cos x} dx &= \int \frac{1}{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}} \frac{2}{1+t^2} dt \\
&= \int \frac{2}{2t+1-t^2} dt \\
&= -2 \int \frac{1}{(t-1-\sqrt{2})(t-1+\sqrt{2})} dt \\
&= -2 \int (\frac{1}{2\sqrt{2}(t-1-\sqrt{2})} - \frac{1}{2\sqrt{2}(t-1+\sqrt{2})}) dt \\
&= \frac{1}{\sqrt{2}} \log|t-1+\sqrt{2}| - \frac{1}{\sqrt{2}} \log|t-1-\sqrt{2}| + C \\
&= \frac{1}{\sqrt{2}} \log|\tan x/2 - 1 + \sqrt{2}| - \frac{1}{\sqrt{2}} \log|\tan x/2 - 1 - \sqrt{2}| + C.
\end{aligned}$$